Periodic Bandits and Wireless Network Selection

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What does this paper do?
We study periodic bandit problems
We study periodic bandit problems

Fully adversarial setting
Define “Periodic Regret”

More general than standard “weak regret” definition
Define “Periodic Regret”

More general than standard “weak regret” definition

But specific enough that EXP4 has a fast implementation on it
Define “Periodic Regret”

More general than standard “weak regret” definition

But specific enough that EXP4 has a fast implementation on it

Fully adversarial approach to periodic problems
Define “Periodic Regret”

Algorithm: Periodic EXP4

\[ O \left( \sqrt{PKT \log K} + KT \log |F| \right) \]
Define “Periodic Regret”

Algorithm: Periodic EXP4

\[ O \left( \sqrt{PKT \log K} + KT \log |F| \right) \]

Periodic Regret Lower Bound

\[ \Omega \left( \sqrt{PKT} + \sqrt{KT \frac{\log |F|}{\log K}} \right) \]
Define “Periodic Regret”

Algorithm: Periodic EXP4

\[ \mathcal{O}\left(\sqrt{PKT\log K} + KT\log|F|\right) \]

\[ \Omega\left(\sqrt{PKT} + \sqrt{KT\frac{\log|F|}{\log K}}\right) \]
Define “Periodic Regret”

Algorithm: Periodic EXP4

Periodic Regret Lower Bound

Robust to less conventional problems (e.g. a distributed setting, network selection)
Multi-Armed Bandits (Adversarial)
Arms

1
2
3
4
Example Run

t=1

1
2
3
4
Example Run

<table>
<thead>
<tr>
<th>t=1</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
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<tbody>
<tr>
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</table>
Example Run

t=1

1

2

0.8

3

4
Example Run

t=1

1

2  0.8

3

4
Example Run

t=2

1

2

3

4

0.8

0.2
Example Run

t=3

1 1.0
2 0.8
3 0.2
4
Example Run

t=5

1.0  0.8

0.8  0.2

0.3  0.8

1  2  3  4
Example Run

$\begin{array}{cccccc}
& & & & & t=6 \\
1 & & & & &
\end{array}$

$\begin{array}{cccccc}
& & & & &
2 & 0.8 & & & 0.6
\end{array}$

$\begin{array}{cccccc}
& & & & &
3 & 0.2 & & &
\end{array}$

$\begin{array}{cccccc}
& & & & &
4 & & & 0.3 & &
\end{array}$
Example Run

\[
\begin{array}{cccc}
1 & 2 & 3 & 4 \\
\text{t=7} & 1.0 & 0.8 & 0.7 \\
0.8 & & & 0.6 \\
0.2 & & 0.3 & \\
\end{array}
\]
|   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   | 1.0 | 0.8 | 0.7 |   |   |   | 0.8 | 0.6 |   |   | 0.2 | 0.3 |   |   |   | {Example Run}
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<tr>
<td>4</td>
<td></td>
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<td>0.3</td>
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Total Reward: 4.4
Total Reward: 4.4
Total Reward: 4.4

How well did the algorithm perform?
Total Reward: 4.4

How well did the algorithm perform?
Regret
Regret

Comparison with how well you “could have done”
Regret

\( \text{OPT} - \text{ALG} \)
Regret

$$OPT - ALG$$

Algorithm’s Performance
Regret

$OPT - ALG$

The “best possible” performance
Most commonly used notion of regret:
Most commonly used notion of regret:

Weak Regret

Comparison against best overall arm
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<td>0.5</td>
<td>0.7</td>
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**Weak Regret**
## Weak Regret

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<td>0.7</td>
<td>0.8</td>
<td>0.6</td>
<td>0.3</td>
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</tbody>
</table>

### OPT

- Position 2
- Value: 0.8, 0.7, 0.9, 0.8, 1.0, 0.6, 0.8

### Weak Regret

- Position 1
- Value: 0.6, 0.7, 1.0, 0.8, 0.5, 0.7, 0.7

- Position 4
- Value: 0.7, 0.8, 0.6, 0.6, 0.3, 0.4, 0.6
Weak Regret

Best Arm: 5.6
Weak Regret

Actual: 4.4

Best Arm: 5.6

1

0.6
0.7
1.0
0.8
0.5
0.7
0.7

2

0.8
0.7
0.9
0.8
1.0
0.6
0.8

3

0.4
0.2
0.6
0.4
0.6
0.9
0.4

4

0.7
0.8
0.6
0.6
0.3
0.4
0.6
Weak Regret = 1.2
Actual: 4.4
Best Arm: 5.6
Weak Regret = 1.2

Actual: 4.4

Best Arm: 5.6

\( O(\sqrt{KT}) \)
Weak Regret = 1.2

Actual: 4.4

Best Arm: 5.6

\[ \Omega(\sqrt{KT}) \]

\[ O(\sqrt{KT}) \]
What about the “actual” optimal strategy?
Full Regret = 2.7

Actual: 4.4

Best: 6.1
Full Regret = 2.7

Actual: 4.4
Best: 6.1

Ω(T)
But does being competitive with a constant policy really mean your algorithm is good?
Periodic Patterns
Periodic Patterns
Periodic Patterns

Arm Score = 4

1 1 0 1 0 1 0 1 0 1 0

2 0 1 0 1 0 1 0 1 0 1
Periodic Patterns

Arm Score = 4
Periodic Patterns

Score = 4

Arm Score = 4
Periodic Patterns
Weak Regret = 0

Score = 4

Arm Score = 4
Periodic Patterns
Weak Regret = -4

Score = 8

Arm Score = 4
Periodic Patterns

1 1 0 0 0 0 1 0 0 0 0 0

2 0 1 0 0 0 0 1 0 0 0 0

3 0 0 1 0 0 0 0 0 1 0

4 0 0 0 1 0 0 0 0 1
Periodic Patterns

1 0 0 0 0 1 0 0 0

0 1 0 0 0 0 1 0 0

0 0 1 0 0 0 0 1 0

0 0 0 1 0 0 0 0 1
Periodic Patterns

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Each row represents a periodic pattern, with the highlighted squares indicating the pattern's continuation.
### Periodic Patterns

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Periodic Patterns

1 0 0 0 0 1 0 0 0
0 1 0 0 0 0 1 0 0
0 0 1 0 0 0 0 1 0
0 0 0 1 0 0 0 0 1
Periodic Regret
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Periodic Regret

1

2

1

2

1

2

1
Periodic Regret

1 1 1 1
0 0 0 0

1 0 0 0
1 1 1 1

2 0 0 0
2 1 1 1
Periodic Regret

Arm Score = 8
We can represent this with labels
Periodic Regret

Label

1 0 1 0 1 0 1 0 1 1 0

2 0 1 0 1 0 1 0 1 0 1
Different Periodic Patterns

Rule:

OPT must play the same arm on time steps which have the same label.
Different Periodic Patterns
Different Periodic Patterns

\[ f_1 \]

\[ A \, B \, A \, B \, A \, B \, A \, B \, A \, B \, A \, B \, A \]
Different Periodic Patterns

\[ f_2 \]

\[
\begin{array}{cccccccc}
\hline
1 & & & & & & & \\
2 & & & & & & & \\
3 & & & & & & & \\
\end{array}
\]
Different Periodic Patterns

\[ f_3 \]

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Different Periodic Patterns

\[ f_4 \]

1

2

3
Different Periodic Patterns

Note: You don’t need to pick a different arm for each label

\[ f_4 \]

\[
\begin{array}{ccccccccc}
\text{A} & \text{A} & \text{A} & \text{A} & \text{B} & \text{B} & \text{B} & \text{B} & \text{C} & \text{C} & \text{C} & \text{C} \\
\end{array}
\]
Different Periodic Patterns

Note: You don’t need to pick a different arm for each label

\[ f_4 \]

\[
\begin{array}{cccccccc}
C & C & C & C & & & & \\
\end{array}
\]
Different Periodic Patterns

Note: You don’t need to pick a different arm for each label

\[ f_4 \]

1

2

3

A A A A B B B B C C C

1

2

3
One periodic pattern:
Easy to solve
Solving ABABABABABAB

Algorithm

```
1 1 1 1 1
0 0 0 0 0
0 0 0 0 0
1 1 1 1 1
2 2 2 2 2
```
Solving ABABABABABABAB

1

2

1

0

0

1

0

0

0

1

0

1

0

1

0

1

1
Solving ABABABABABABAB

Because the algorithm knows the period
Different Periodic Patterns

In general, we deal with multiple possible periodic patterns at once
Different Periodic Patterns

\[
\begin{align*}
  f_1 & : \quad \text{A B A B A B A B A B A B A B} \\
  f_2 & : \quad \text{A B C A B C A B C A B C} \\
  f_3 & : \quad \text{A A B C B A A B C B A A B C B} \\
  f_4 & : \quad \text{A A A A A A B B B B B B C C C} \\
\end{align*}
\]
Different Periodic Patterns

“OPT” is allowed to pick any one of the label functions and play according to that pattern.
Different Periodic Patterns

\[
\begin{align*}
  f_1 & : A \ B \ A \ B \ A \ B \ A \ B \ A \ B \ A \ B \ A \\
  f_2 & : A \ B \ C \ A \ B \ C \ A \ B \ C \ A \ B \ C \ A \\
  f_3 & : A \ A \ B \ C \ B \ A \ A \ B \ C \ B \ A \ A \\
  f_4 & : A \ A \ A \ A \ A \ A \ B \ B \ B \ B \ B \ C \ C \ C
\end{align*}
\]
### Different Periodic Patterns

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<th>B</th>
<th>A</th>
<th>B</th>
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### Table 1

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### Table 3

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Solving with EXP4
EXP4:
Solves the "Bandits with Expert Advice" problem
EXP4 actually solves a more general problem.
EXP4 actually solves a more general problem.

We have a set of experts. Each expert recommends an arm at each time step.
EXP4 actually solves a more general problem.

We have a set of experts. Each expert recommends an arm at each time step.

One Expert:
EXP4: competitive with best expert in the set
$f_3$  

\[ \begin{array}{cccccccccc}
& & & & & & & & & \\
& & & & & & & & & \\
& & & & & & & & & \\
& & & & & & & & & \\
& & & & & & & & & \\
& & & & & & & & & \\
\end{array} \]
This label function
This label function can be modeled with $K^3$ experts.
Can be modeled with $K^3$ experts
Regret bound: $\sqrt{KT\log|\Pi|}$
Regret bound: $\sqrt{KT\log|\Pi|}$

$K = \text{number of arms}$

$F$ set of label functions

$\Pi$ set of experts
Regret bound: \( \sqrt{KT \log |\Pi|} \)

\( F \) set of label functions

\( \Pi \) set of experts

\( T = \) number of time steps

\( K = \) number of arms
$F$ set of label functions

$\Pi$ set of experts

\[ |\Pi| = \sum_{f \in F} K^{\text{numlabels}(f)} \]

Regret bound: \[ \sqrt{KT\log|\Pi|} \]
$\Pi$ set of experts

$F$ set of label functions

$|\Pi| \leq |F| K^p$

Regret bound: $\sqrt{KT \log |\Pi|}$
Regret bound: $\sqrt{KT\log(|F|K^p)}$
Regret bound: $\sqrt{KT\log(|F|K^P)}$

$O \left( \sqrt{PKT\log K + KT\log|F|} \right)$
Regret bound: \( \sqrt{KT \log(|F|K^P)} \)

\( \Theta \left( \sqrt{PKT \log K + KT \log |F|} \right) \)

**Good**
Issue with EXP4:
Issue with EXP4:
It’s slow
Issue with EXP4:
It’s slow
Stores weights for $|\Pi|$ experts
Issue with EXP4:
It’s slow
Stores weights for $|\Pi|$ experts
Computes weights $|\Pi|$ experts
|\Pi| \simeq |F| K^P

Stores weights for \(|\Pi|\) experts
Computes weights \(|\Pi|\) experts
\[ |\Pi| \sim |F| K^P \]

Stores weights for \(|\Pi|\) experts
Computes weights for \(|\Pi|\) experts
Turns out that the periodic problem is special
Turns out that the periodic problem is special
Turns out that the periodic problem is special

timestep $t$

EXP4
Turns out that the periodic problem is special

timestep $t$

all expert weights

$\Pi$

$|F|^p$
Turns out that the periodic problem is special

timestep $t$

all expert weights

$\Pi$

$|F|K^p$

$K$ arm probabilities

$p_1$

$p_2$

$p_3$

$p_4$
Turns out that the periodic problem is special
Turns out that the periodic problem is special.

Exploit Symmetries in Periodic Problem

A periodic problem has $K$ arm probabilities $p_1, p_2, p_3, p_4$.

The timestep $t$ is the point of interest in this periodic problem.
Turns out that the periodic problem is special.

Exploit Symmetries in Periodic Problem

Computes the exact same probabilities with polynomial time/space
How?
How?

Idea:
when algo plays Arm 3 on a Label B timestep,
only the experts that play Arm 3 on label B will be updated.
\[ t = 1, 2, 3 \quad t = 4, 5 \quad t = 6, 7, 8 \]
\[ \text{label} = A \quad \text{label} = B \quad \text{label} = C \]

\[
\begin{align*}
b_1^A &= 1 & b_1^B &= 1 & b_1^C &= 1 \\
b_2^A &= 1 & b_2^B &= 1 & b_2^C &= 1 \\
b_3^A &= 1 & b_3^B &= 1 & b_3^C &= 1 \\
b_4^A &= 1 & b_4^B &= 1 & b_4^C &= 1
\end{align*}
\]
\[ t = 1, 2, 3 \quad t = 4, 5 \quad t = 6, 7, 8 \quad \text{next step: label } B \]

\[ \text{label } = A \quad \text{label } = B \quad \text{label } = C \]

\[
\begin{align*}
 b_1^A &= e_3 & b_1^B &= 1 & b_1^C &= 1 \\
 b_2^A &= e_1 & b_2^B &= 1 & b_2^C &= e_6e_8 \\
 b_3^A &= 1 & b_3^B &= e_4e_5 & b_3^C &= 1 \\
 b_4^A &= e_2 & b_4^B &= 1 & b_4^C &= e_7
\end{align*}
\]
$t = 1, 2, 3 \quad t = 4, 5 \quad t = 6, 7, 8 \quad \text{next step:}$

$\text{label} = A \quad \text{label} = B \quad \text{label} = C \quad \text{label} B$

\[ b_1^A = e_3 \quad b_1^B = 1 \quad b_1^C = 1 \]
\[ b_2^A = e_1 \quad b_2^B = 1 \quad b_2^C = e_6 e_8 \]
\[ b_3^A = 1 \quad b_3^B = e_4 e_5 \quad b_3^C = 1 \]
\[ b_4^A = e_2 \quad b_4^B = 1 \quad b_4^C = e_7 \]

\[ e_t = \exp \left( \frac{\gamma}{K} \hat{x}_{it}(t) \right) \]

**algorithm’s plays**
\[ t = 1, 2, 3 \quad t = 4, 5 \quad t = 6, 7, 8 \quad \text{next step: label B} \]

\[ \text{label} = A \quad \text{label} = B \quad \text{label} = C \]

\[
\begin{align*}
  b_1^A &= e_3 & b_1^B &= 1 & b_1^C &= 1 \\
  b_2^A &= e_1 & b_2^B &= 1 & b_2^C &= e_6 e_8 \\
  b_3^A &= 1 & b_3^B &= e_4 e_5 & b_3^C &= 1 \\
  b_4^A &= e_2 & b_4^B &= 1 & b_4^C &= e_7
\end{align*}
\]

combined weight adjustments
weight adjustment for playing Arm 1 on Label A

\[ t = 1, 2, 3 \quad t = 4, 5 \quad t = 6, 7, 8 \quad \text{next step:} \]

\[ \text{label } = A \quad \text{label } = B \quad \text{label } = C \quad \text{label } B \]

\[
\begin{align*}
 b^A_1 &= e_3 \\
 b^A_2 &= e_1 \\
 b^A_3 &= 1 \\
 b^A_4 &= e_2 \\
 b^B_1 &= 1 \\
 b^B_2 &= 1 \\
 b^B_3 &= e_4 e_5 \\
 b^B_4 &= 1 \\
 b^C_1 &= 1 \\
 b^C_2 &= e_{68} \\
 b^C_3 &= 1 \\
 b^C_4 &= e_7
\end{align*}
\]
weight adjustment for playing Arm 2 on Label A

\[
\begin{align*}
  b_1^A &= e_3, & b_1^B &= 1, & b_1^C &= 1 \\
  b_2^A &= e_1, & b_2^B &= 1, & b_2^C &= e_8 \\
  b_3^A &= 1, & b_3^B &= e_4 e_5, & b_3^C &= 1 \\
  b_4^A &= e_2, & b_4^B &= 1, & b_4^C &= e_7
\end{align*}
\]
For playing Arm 3 on Label A:

\[ b_1^A = e_3 \]
\[ b_2^A = e_1 \]
\[ b_3^A = 1 \]
\[ b_4^A = e_2 \]

For playing Arm 3 on Label B:

\[ b_1^B = 1 \]
\[ b_2^B = 1 \]
\[ b_3^B = e_4 e_5 \]
\[ b_4^B = 1 \]
\[ b_3^C = 1 \]
\[ b_4^C = e_7 \]
weight adjustment for playing Arm 3 on Label B
Now I want to compute the probabilities for the next time step

\[ b_1^A = e_3 \quad b_1^B = 1 \quad b_1^C = 1 \]

\[ b_4^A = e_2 \quad b_4^B = 1 \quad b_4^C = e_7 \]
\[
\begin{align*}
 t = 1, 2, 3 & \quad t = 4, 5 & \quad t = 6, 7, 8 & \quad \text{next step:} & \quad \text{label } B \\
\text{label } A & \quad \text{label } B & \quad \text{label } C \\
\end{align*}
\]

\[
\begin{array}{c}
\begin{array}{cccc}
1 & 2 & 3 & 4 \\
e_1 & e_2 & e_3 & \text{ } \text{ } \text{ } \\
e_4 & e_5 & e_6 & e_7 \\
e_8 & \text{ } \text{ } \text{ } & \text{ } \text{ } \\
\end{array}
\end{array}
\]

\[
\begin{align*}
b_1^A &= e_3 & b_1^B &= 1 & b_1^C &= 1 \\
b_2^A &= e_1 & b_2^B &= 1 & b_2^C &= e_6 e_8 \\
b_3^A &= 1 & b_3^B &= e_4 e_5 & b_3^C &= 1 \\
b_4^A &= e_2 & b_4^B &= 1 & b_4^C &= e_7 \\
\end{align*}
\]
example: compute probability for Arm 3

\[ b_2^A = e_1 \quad b_2^B = 1 \quad b_2^C = e_6 e_8 \]

\[ b_3^A = 1 \quad b_3^B = e_4 e_5 \quad b_3^C = 1 \]

\[ b_4^A = e_2 \quad b_4^B = 1 \quad b_4^C = e_7 \]
example: compute probability for Arm 3

\[ b_2 = \] 

\[ b_3^A = \] 

\[ b_4^A = e_2 \quad b_4^B = 1 \quad b_4^C = e_7 \]
\[ t = 1, 2, 3 \quad t = 4, 5 \quad t = 6, 7, 8 \quad \text{next step: label } B \]

\[ \text{label } = A \quad \text{label } = B \quad \text{label } = C \]

\[ b_1^A = e_3 \quad b_1^B = 1 \quad b_1^C = 1 \]
\[ b_2^A = e_1 \quad b_2^B = 1 \quad b_2^C = e_6 e_8 \]
\[ b_3^A = 1 \quad b_3^B = e_4 e_5 \quad b_3^C = 1 \]
\[ b_4^A = e_2 \quad b_4^B = 1 \quad b_4^C = e_7 \]
$t = 1, 2, 3$  
$label = A$

$t = 4, 5$  
$label = B$

$t = 6, 7, 8$  
$label = C$

next step:  
$label B$

\[
\begin{align*}
 b^A_1 &= e_3 \\
 b^B_1 &= 1 \\
 b^C_1 &= 1 \\
 b^A_2 &= e_1 \\
 b^B_2 &= 1 \\
 b^C_2 &= e_6 e_8 \\
 b^A_3 &= 1 \\
 b^B_3 &= e_4 e_5 \\
 b^C_3 &= 1 \\
 b^A_4 &= e_2 \\
 b^B_4 &= 1 \\
 b^C_4 &= e_7 \\
 b^A_1 b^B_3 b^C_1
\end{align*}
\]
\[ b_1^A = e_3 \quad b_1^B = 1 \quad b_1^C = 1 \]
\[ b_2^A = e_1 \quad b_2^B = 1 \quad b_2^C = e_6 e_8 \]
\[ b_3^A = 1 \quad b_3^B = e_4 e_5 \quad b_3^C = 1 \]
\[ b_4^A = e_2 \quad b_4^B = 1 \quad b_4^C = e_7 \]
\[ t = 1, 2, 3 \quad t = 4, 5 \quad t = 6, 7, 8 \quad \text{next step:} \]

**label = A** \quad **label = B** \quad **label = C** \quad **label B**

\[
\begin{align*}
&b_1^A = e_3 & b_1^B = 1 & b_1^C = 1 \\
&b_2^A = e_1 & b_2^B = 1 & b_2^C = e_6e_8 \\
&b_3^A = 1 & b_3^B = e_4e_5 & b_3^C = 1 \\
&b_4^A = e_2 & b_4^B = 1 & b_4^C = e_7
\end{align*}
\]
\[ t = 1, 2, 3 \quad t = 4, 5 \quad t = 6, 7, 8 \quad \text{next step:} \quad \text{label } B \]

**Label A**

- \( b_1^A = e_3 \)
- \( b_2^A = e_1 \)
- \( b_3^A = 1 \)
- \( b_4^A = e_2 \)

**Label B**

- \( b_1^B = 1 \)
- \( b_2^B = 1 \)
- \( b_3^B = e_4 e_5 \)
- \( b_4^B = 1 \)

**Label C**

- \( b_1^C = 1 \)
- \( b_2^C = e_6 e_8 \)
- \( b_3^C = 1 \)
- \( b_4^C = e_7 \)

\[ b_1^A b_3^B b_1^C \]
\[ b_1^A b_3^B b_2^C \]
\[ b_1^A b_3^B b_3^C \]
\[ b_1^A b_3^B b_4^C \]
\[ t = 1, 2, 3 \quad label = A \]
\[ t = 4, 5 \quad label = B \]
\[ t = 6, 7, 8 \quad label = C \]

next step: label B

\[
\begin{align*}
    b_1^A &= e_3 \\
    b_2^A &= e_1 \\
    b_3^A &= 1 \\
    b_4^A &= e_2 \\
    b_1^B &= 1 \\
    b_2^B &= 1 \\
    b_3^B &= e_4 e_5 \\
    b_4^B &= 1 \\
    b_1^C &= 1 \\
    b_2^C &= e_6 e_8 \\
    b_3^C &= 1 \\
    b_4^C &= e_7
\end{align*}
\]
Add these together to compute weight

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<td>$b_3^B$</td>
<td>$b_1^C$</td>
<td>$b_2^A$</td>
<td>$b_3^B$</td>
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</table>
Add these together to compute weight

\[ b_1^A b_3^B b_1^C + b_2^A b_3^B b_2^C + b_3^A b_3^B b_1^C + b_4^A b_3^B b_1^C \]

\[ b_1^A b_3^B b_2^C + b_2^A b_3^B b_2^C + b_3^A b_3^B b_2^C + b_4^A b_3^B b_2^C \]

\[ b_1^A b_3^B b_3^C + b_2^A b_3^B b_3^C + b_3^A b_3^B b_3^C + b_4^A b_3^B b_3^C \]

\[ b_1^A b_3^B b_4^C + b_2^A b_3^B b_4^C + b_3^A b_3^B b_4^C + b_4^A b_3^B b_4^C \]
Add these together to compute weight

\[
b_1^A b_3^B b_1^C + b_2^A b_3^B b_2^C + b_3^A b_3^B b_1^C + b_4^A b_3^B b_1^C
\]

\[
b_1^A b_3^B b_2^C + b_2^A b_3^B b_2^C + b_3^A b_3^B b_2^C + b_4^A b_3^B b_2^C
\]

\[
b_1^A b_3^B b_3^C + b_2^A b_3^B b_3^C + b_3^A b_3^B b_3^C + b_4^A b_3^B b_3^C
\]

\[
b_1^A b_3^B b_4^C + b_2^A b_3^B b_4^C + b_3^A b_3^B b_4^C + b_4^A b_3^B b_4^C
\]

\[
b_3^B (b_1^A + b_2^A + b_3^A + b_4^A)(b_1^C + b_2^C + b_3^C + b_4^C)
\]
Running Time

Memory: $O(KP|F|)$

Running Time: $O(K^2P|F|)$
Running Time

Memory: \( O(KP|F|) \)

Running Time: \( O(K^2P|F|T) \)  
\( O(KP|F| + K|F|T) \)
Periodic Regret

More general than standard “weak regret” definition

But specific enough that EXP4 has a fast implementation on it

Fully adversarial approach to periodic problems
Periodic Regret

Fully adversarial approach to periodic problems
Periodic Regret

Makes no assumption on the adversary at all

Fully adversarial approach to periodic problems
Periodic Regret

Makes no assumption on the adversary at all

Problem is defined only by changing the regret measure

Fully adversarial approach to periodic problems
Lower Bounds
Weak Regret Lower Bound

\[ \Omega(\sqrt{KT}) \]
Weak Regret Lower Bound

$\Theta(\sqrt{KT})$

$K \leq T$
Regret for one label function with $P$ labels
Regret for one label function with $P$ labels

$f$

\[ \frac{T}{P}, \frac{2T}{P}, \frac{3T}{P}, \ldots \]

\[ \begin{array}{ccccccc}
A & B & C & \cdots & \\
1 & \frac{T}{P} & \frac{T}{P} & \frac{T}{P} & \cdots & T
\end{array} \]
Regret for one label function with $P$ labels

$$\Theta \left( P \times \sqrt{K \frac{T}{P}} \right)$$
Regret for one label function with $P$ labels

$\Theta\left(\sqrt{PKT}\right)$
Q: Does having more than one label function make the problem harder?
Q: Does having more than one label function make the problem harder?

A: Yes
\[ P = \text{Max. number of labels on any label function} \]
\[ K = \text{Number of arms} \]
\[ |F| = \text{Number of label functions} \]
\[ P = \text{Max. number of labels on any label function} \]

\[ K = \text{Number of arms} \]

\[ |F| = \text{Number of label functions} \]

**Assume** \( K \leq P \)
Given: $P = \text{Max. number of labels on any label function}$

$K = \text{Number of arms}$

$|F| = \text{Number of label functions}$

Assume $K \leq P$
Define some $M \geq P$
\( P \) = Max. number of labels on any label function

\( K \) = Number of arms

\( |F| \) = Number of label functions

\( F := \) All partitions of the set \( \{1, 2, \ldots, M\} \) into \( K \) parts

\( \Omega(\sqrt{MKT}) \)
\( P = \) Max. number of labels on any label function

\( K = \) Number of arms

\( |F| = \) Number of label functions

\[ F := \text{All partitions of the set } \{1, 2, \ldots, M\} \text{ into } K \text{ parts} \]

Express \( M \) in terms of \(|F|\)

\[ \Omega(\sqrt{MKT}) \]
\[ P = \text{Max. number of labels on any label function} \]
\[ K = \text{Number of arms} \]
\[ |F| = \text{Number of label functions} \]

\[
\Omega\left(\sqrt{KT \frac{\log |F|}{\log K}}\right)
\]
\[ P = \text{Max. number of labels on any label function} \]
\[ K = \text{Number of arms} \]
\[ |F| = \text{Number of label functions} \]

\[ \Omega\left(\sqrt{P KT}\right) \quad \Omega\left(\sqrt{KT \frac{\log|F|}{\log K}}\right) \]

Add one more label function to \( F \)

\[ 1 \quad 2 \quad 3 \quad 4 \quad \ldots \quad M \]
\( P = \) Max. number of labels on any label function
\( K = \) Number of arms
\( |F| = \) Number of label functions

\[
\Omega \left( \sqrt{P KT} + \sqrt{KT \frac{\log |F|}{\log K}} \right)
\]
Lower Bound:

$$\Omega \left( \sqrt{P KT} + \sqrt{K T \frac{\log |F|}{\log K}} \right)$$
Lower Bound:

$$\Omega \left( \sqrt{\frac{PKT}{|F|}} + \sqrt{KT \frac{\log|F|}{\log K}} \right)$$

Upper Bound:

$$O \left( \sqrt{PKT \log K + KT \log|F|} \right)$$
Lower Bound:

$$
\Omega \left( \sqrt{PKT} + \sqrt{KT \frac{\log|F|}{\log K}} \right)
$$

Upper Bound:

$$
O \left( \sqrt{PKT \log K} + KT \log|F| \right)
$$
Wireless Network Selection
Periodic EXP4
Periodic EXP4
Periodic EXP4

Periodic Regret Bound

\( O \left( \sqrt{P KT \log K + KT \log |F|} \right) \)
Network Selection Model
users
users

networks

24

14

6
time step 1
users  networks

networks?
users

networks

24

14

6
users

networks

?
users  networks

e tc...
users

networks

?
users

networks
users

Periodic EXP4

networks

arms

24

14

6
Problem Instances
Example: Discrete

![Graph showing data rate over time slots for different network types. The graph displays three lines representing Cellular, Office WiFi 1, and Office WiFi 2. The x-axis represents time slots, and the y-axis represents data rate in Mbps. The graph indicates fluctuations in data rate for each network type.]
Example: Discrete
Example: Discrete

Cellular -- Orange WiFi 1 -- Orange WiFi 2

Time slot

One Day
Example: Discrete
Example: Discrete

One Day
Example: Discrete

Known best period: 4
Example: Discrete

Example: Continuous
**Example: Discrete**

![Graph showing discrete data rates]

**Example: Continuous**

![Graph showing continuous data rates]
No clear “best” period

Example: Continuous
Label Functions for Periodic EXP4
Label Functions for Periodic EXP4 something simple.
One Day

\[ f_1 \]

\[ A \]
One Day

\[ f_1 \]

\[ f_2 \]
One Day

$f_1$  

$f_2$  

$f_3$  

A A A

B B B

C C C
One Day

\( f_1 \)  
\( f_2 \)  
\( f_3 \)  
\( f_4 \)  
\( f_5 \)
Multiple days

$f_4$
Does it learn?
Discrete Instance

Continuous Instance
Periodic EXP4’s performance improves with time
Contrast: EXP3 doesn’t learn
“Ground Truth” (Bandwidth Ratio)
“Ground Truth” (Bandwidth Ratio)
EXP3

Total arm probabilities

slowly averages out...
EXP3 is trying to find a “good” constant choice of arm.
Periodic EXP4

“converging” to the bandwidth ratios
Periodic EXP4

$converging$ to the bandwidth ratios
Periodic EXP4

"converging" to the bandwidth ratios
Periodic EXP4 "converging" to the bandwidth ratios
Periodic EXP4

“converging” to the bandwidth ratios

Periodic EXP4 is trying to find a “good” periodic choice of arm.
Periodic Regret

More general than standard “weak regret” definition
Periodic Regret

More general than standard “weak regret” definition

Has regret bounds which are “tight” up to a $\log K$ factor with an efficient algorithm
Periodic Regret

More general than standard “weak regret” definition

Has regret bounds which are “tight” up to a \( \log K \) factor with an efficient algorithm

Fully adversarial approach to periodic problems
Periodic Regret

More general than standard “weak regret” definition

Has regret bounds which are “tight” up to a $\log K$ factor with an efficient algorithm

Fully adversarial approach to periodic problems

Robust to less conventional problems (e.g. distributed setting)
Open Questions / Future Work
What is a good choice of label functions for specific problems?
Open Questions / Future Work

What is a good choice of label functions for specific problems?

Comparison between adversarial and stochastic approaches to periodic problems
Open Questions / Future Work

What is a good choice of label functions for specific problems?

Comparison between adversarial and stochastic approaches to periodic problems

Closing the $\log N$ gap between the upper/lower regret bounds
Weak regret

\[ O(\sqrt{KT}) \]
\[ \Omega(\sqrt{KT}) \]

Regret vs Experts

\[ O(\sqrt{KT\log|\Pi|}) \]
\[ \Omega(\sqrt{KT\log|\Pi|/\log K}) \]

Periodic Regret

\[ O\left(\sqrt{PKT\log K + KT\log|F|}\right) \]
\[ \Omega\left(\sqrt{PKT} + \sqrt{KT\log|F|/\log K}\right) \]
But we're not reaching 0% loss?
But we’re not reaching 0% loss?
“Optimal Random”

users  networks

24  24/44 \approx 54.5\%

14  14/44 \approx 31.8\%

6  6/44 \approx 13.6\%
“Optimal Random”

users

54.5%  24
31.8%  14
13.6%  6
Converges to optimal distribution
Learning the "correct" distribution
users

arm probabilities

24

14

6
users

add up the arm probabilities
users

and you get a total distribution
The “correct distribution”
(actually the ratio of the bandwidths)
The “correct distribution” (actually the ratio of the bandwidths)
EXP3

slowly averages out...
**EXP3**

slowly averages out...

**EXP3** is trying to find a “good” constant choice of arm.
Periodic EXP4

"converging" to the bandwidth ratios
Periodic EXP4

“converging” to the bandwidth ratios
Periodic EXP4

“converging” to the bandwidth ratios
Periodic EXP4 converging to the bandwidth ratios.
Periodic EXP4 is trying to find a "good" periodic choice of arm. "converging" to the bandwidth ratios

Periodic EXP4 is trying to find a "good" periodic choice of arm.
Performance Metric
Look at the “worst scoring user”
Look at the “worst scoring user”
Compare with the lowest scoring user in the "best allocation"
Compare with the lowest scoring user in the “best allocation”
Compare with the lowest scoring user in the "best allocation"
Take the loss as a ratio:
Take the loss as a ratio:

$$\frac{6}{2}$$
Take the loss as a ratio:

\[
\frac{6 - 2}{6}
\]
Take the loss as a ratio:

\[
\frac{6 - 2}{6} \approx 66.7\% \text{ loss}
\]